



By Catalina Liu
Dulwich College Beijing

Meet the Clan! ☺

Bluestar:

Bluestar is very friendly and kind.

She has a pair of shiny blue eyes!

Lucky:

Lucky is a very smart cat.

She has a pair of green and yellow eyes!

Paws:

Paws is a very lazy cat that has a pair

of yellow eyes!

Sparkle:

Sparkle is an energetic cat. She loves

Sports! She has a pair of purple eyes!

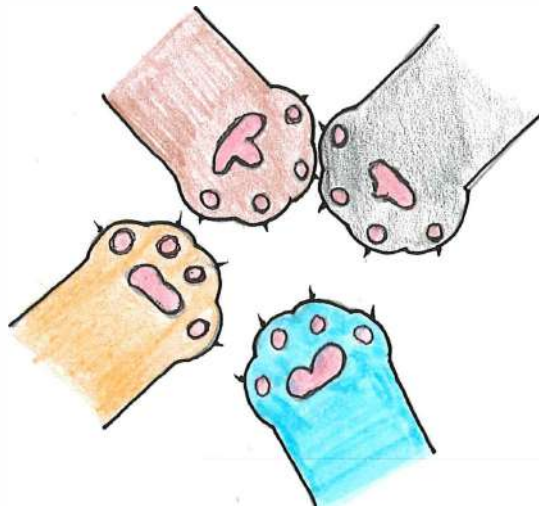
Sparkle: "We are the Moonlight Clan!"

Bluestar: "Today we are going to make the biggest area of our garden!"

Lucky: " I will make the biggest garden!"

Paws: "Let me sleep for a while..... "

Miaow!
Miaow!
Miaow!
Miaow!



Let's see who will get the biggest area of their garden!

"As you know, I am getting older and I would want the following cats: Lucky, Paws, Bluestar and Sparkle to come forward to complete a challenge in order to replace me as a queen. The winning cat will be the next in line to the throne of our cat kingdom. The challenge is to design a garden area for our kittens to play with a wooden fence boundary of a length that is equal to my age of 56 years. The perimeter must be 56m. In my kingdom you are only able to buy 1m sections of fence at a time.

The four cats thought very carefully about this....

"I am going to create a circular garden!" Said Bluestar.

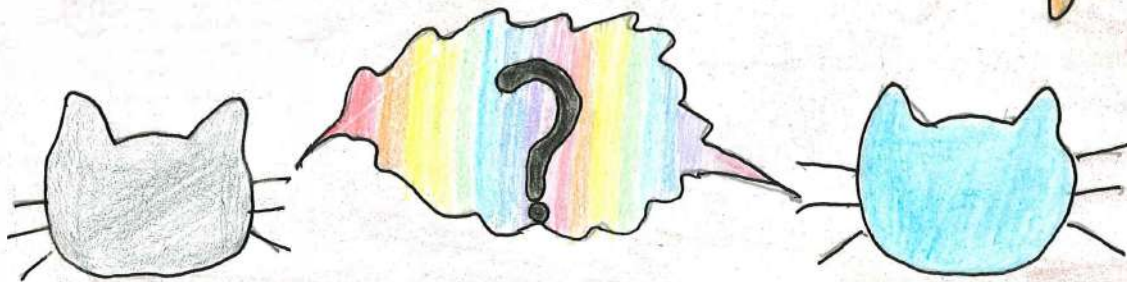
"I will create an isosceles triangle for my garden shape!" Exclaimed Lucky.

"I will make a square shaped garden! No one's going to beat me!" Shouted Paws.

"I will of course do a rectangle, because I think it will be bigger than a square!" Said Sparkle proudly.

Although Paws and Sparkle were born together, they looked different but thought alike.

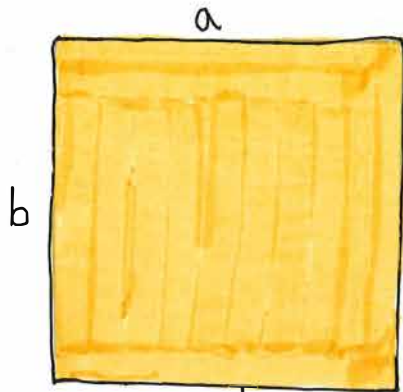
Rectangle vs Square



Paws wanted to do a simple SQUARE, because she thinks a square could take up more area.

To work out the area of this square, Paws needed to work out the side lengths of the square.

She noticed that the side lengths of the square are equal to each other.

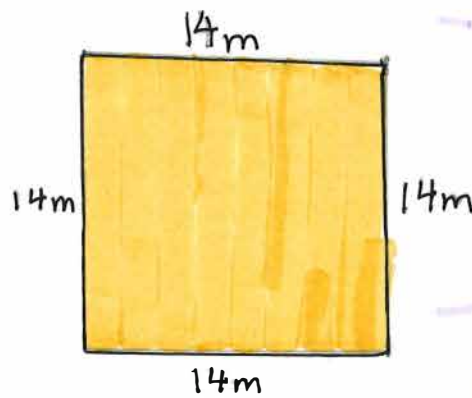


$$a = b = c = d$$



"Oh, that's easy! All I need to do is $56 \div 4 = 14$ meters for each side."

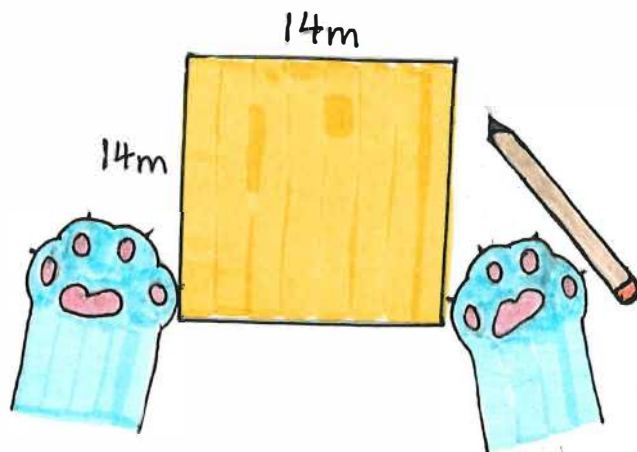
Nice



$$56 \div 4 = 14$$

Check:
 $14 + 14 + 14 + 14 = 56$

Now, to work out the area, Paws need to times 2 side lengths, $14 \times 14 = 196\text{m}^2$.



"Ah! My job is done!"

Sparkle decided to do a rectangle for her garden.

To work out the area of a rectangle, Sparkle has to multiply the length by width.

Now, Sparkle has to think about the measurements of the length and width to make a perimeter of 56m. She thinks about this carefully...

$$2 \times \text{Width} + 2 \times \text{Length} = \text{Perimeter}$$

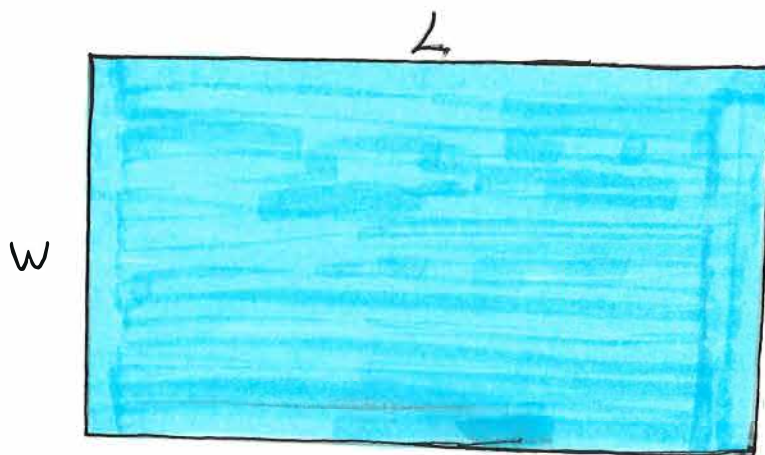
$$2W + 2L = 56 \text{ meters}$$

Wait a moment... I can approach this in a much more, simpler way and that is down to my knowledge of Algebra. Cool eh!

So, $W + L$ will give me half of the perimeter!

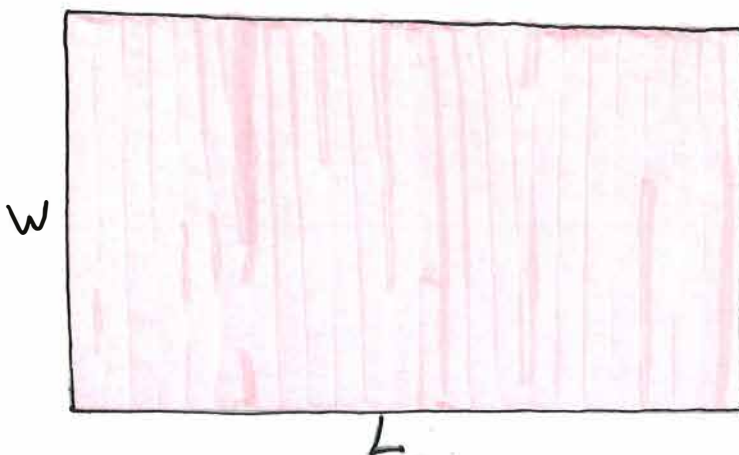
$$\text{So, } W + L = 28.$$

My challenge is to make the biggest area using a width and length in whole number dimensions. That was the queen's instructions!



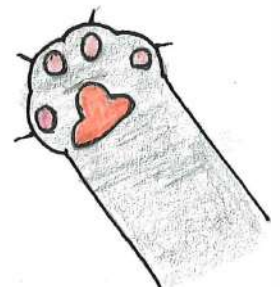
$$2L + 2W = 56m$$

and




$$L + W = 28m$$

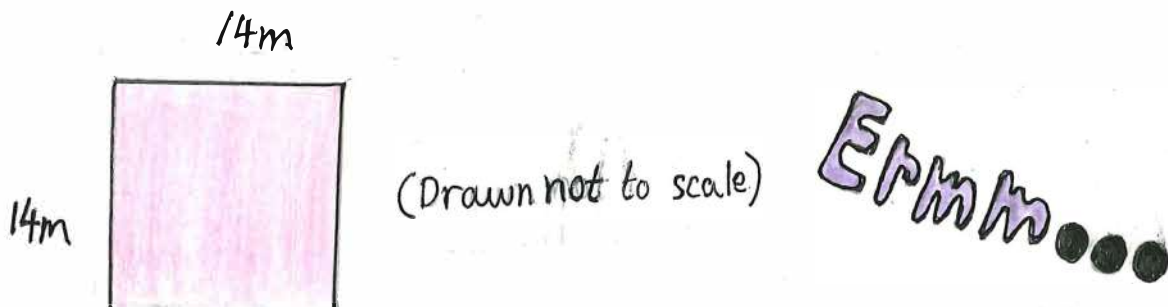
" $2L + 2W$, will give you half of $L + W$!"



So Sparkle thought of a strategic plan. 'Let's List pairs of number whose sum is 28m in an organized way and whose product represents the largest area where the product is the largest number'. Meaning: Length \times Width = Area

1 + 27		$1 \times 27 = 27 \text{ m}^2$
2 + 26		$2 \times 26 = 52 \text{ m}^2$
3 + 25		$3 \times 25 = 75 \text{ m}^2$
4 + 24		$4 \times 24 = 96 \text{ m}^2$
5 + 23		$5 \times 23 = 115 \text{ m}^2$
6 + 22		$6 \times 22 = 132 \text{ m}^2$
7 + 21		$7 \times 21 = 147 \text{ m}^2$
8 + 20		$8 \times 20 = 160 \text{ m}^2$
9 + 19		$9 \times 19 = 171 \text{ m}^2$
10 + 18		$10 \times 18 = 180 \text{ m}^2$
11 + 17		$11 \times 17 = 187 \text{ m}^2$
12 + 16		$12 \times 16 = 192 \text{ m}^2$
13 + 15		$13 \times 15 = 195 \text{ m}^2$
14 + 14		$14 \times 14 = 196 \text{ m}^2$
15 + 13	$15 \times 13 = 195 \text{ m}^2$	
...	...	
...	...	

"Wait a moment, my pairs of numbers are re-appearing but in the reverse order! The next number pair will be 16 + 12 according to my logic. I have discovered that my maximum garden area is when the length and width are equal. 14 m \times 14 m. Hey, wait a moment this is the design for Paw's square garden design. How embarrassing! What will paws think of me now?"



Bluestar had decided to go for the CIRCULAR garden:

To work out the area of his circular garden, Bluestar had to use the formula $Area = \pi r^2$.

There was one problem, Bluestar would need to work out the radius when the perimeter needed to be 56m. In the queen's kingdom you can only purchase sections of fence where measurements are in whole numbers only.

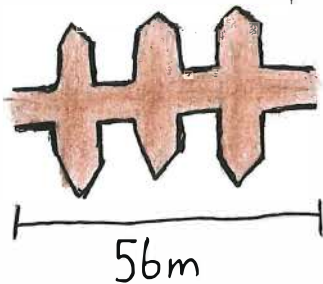
She pondered for a moment scratching her head with her claws...ouch!

Now, If I was given 56 meters of fence, I could work out the radius of this circular garden. I remember learning about this at school. We

take r to be the radius and

π is a special number we use for the following calculation

$\pi = 3.141 \dots$ which continues with a non-repeating sequence of decimal digits!



The perimeter can be calculated by the formula ' $2\pi r$ '

$$2\pi r = 56 \text{ meters}$$

$$\pi r = 28 \text{ meters}$$

$$r = \frac{28}{\pi}$$

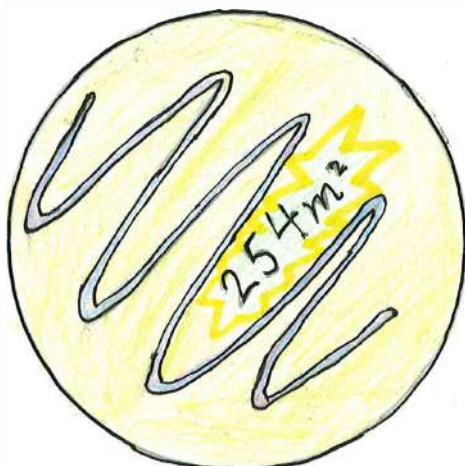
Which makes ' r ' is approximately 9 meters

So Bluestar uses this measurement for calculating his radius ' r '



Bluestar could now calculate the area of his circular garden design = πr^2

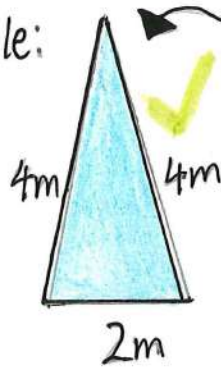
$$\pi \times 9 \times 9 = 254 \text{ m}^2$$



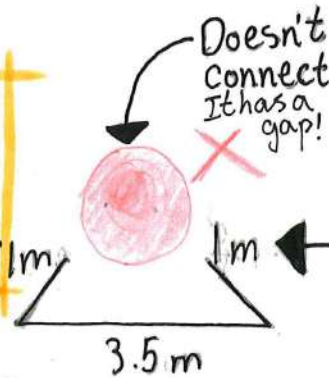
Lucky thought of doing an Isosceles triangle for her garden area. She was so eager to please the queen with her imaginative and logical approach to her garden design whilst rubbing her paws over the thought of owning this wonderful kingdom! She remembered the queen saying that she could only purchase sections of fence of integers measurements of.

"Hmmm..... This requires a little bit logical thinking; an isosceles triangle has two equal sides. The base of the triangle must be smaller than the sum of the 2 equal sides."

For example:



The base is smaller than the 2 sides, this makes an isosceles triangle.

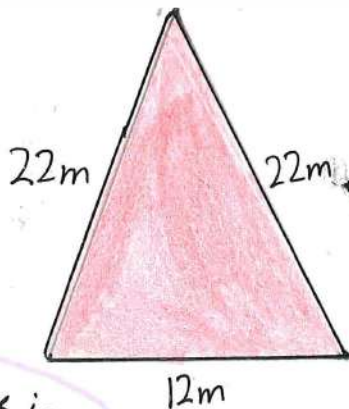


If the sum of the 2 equal sides is smaller than the base, then it can't be a triangle.

(Not to scale)

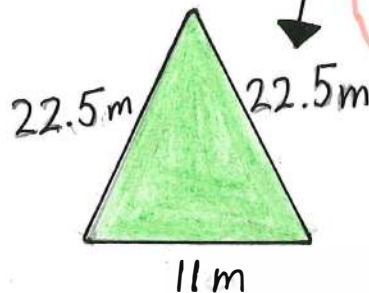
Lucky realised very quickly that the base has to be an even number to ensure that all the sides are integer lengths, and also the base has got to be smaller than the sum of the two equal sides. If the base is bigger than the two equal sides added up, then it will not form a triangle.

(Not to scale)



$$56 - 12 = 44$$

$$44 \div 2 = 22$$



The sides are not integer lengths? No way, the Queen won't allow that!

$$56 - 11 = 45$$

$$45 \div 2 = 22.5$$

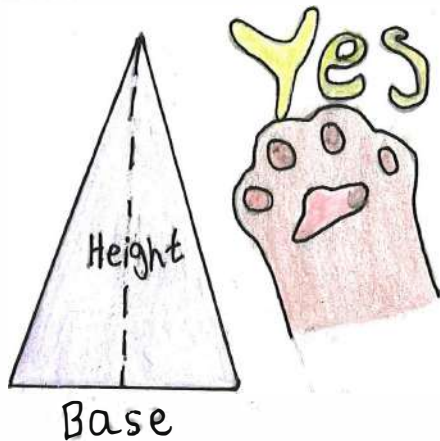
This is a whole integer if the base is even.

Only even numbers
"Don't forget that the Perimeter is 56m!" said Lucky

There is a add number on the base so it won't be whole integers on both side lengths.

Now with all my logical thinking approach, I think I can get creating my Isosceles Triangle Garden design that has the maximum garden area.

'I will first be considering a triangle base of 2m. This will give the two equal sides with a measurement of 27m'. This makes the perimeter of our required triangle to be 56m. Lucky continued this approach of increasing the base in steps of even numbers.

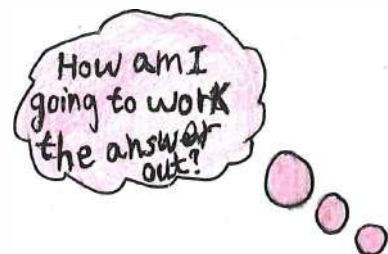
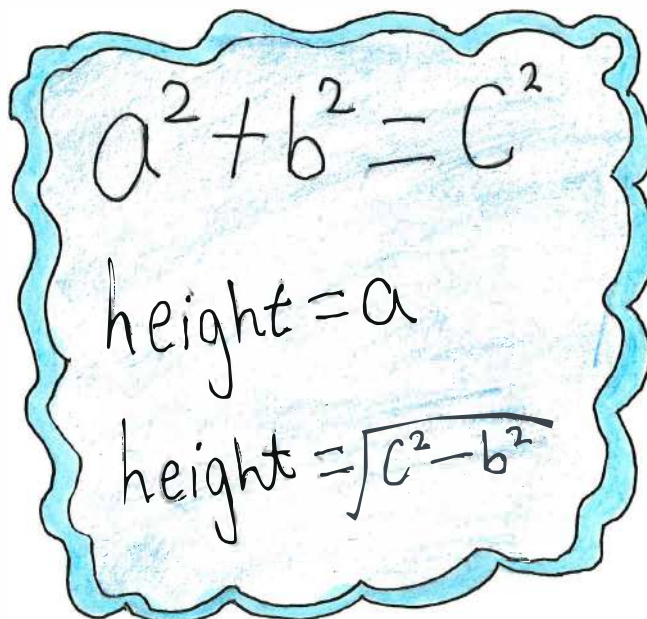
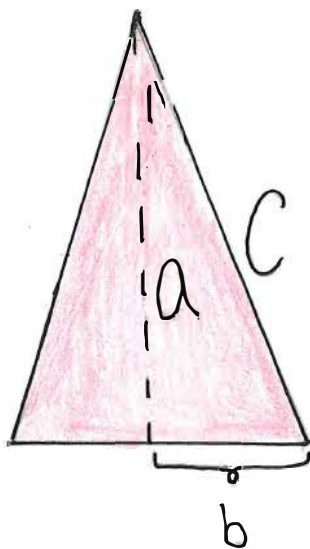


Lucky recalled the formula for calculating the Area of a triangle

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}.$$

The problem was in trying to calculate height.

A Greek Philosopher by the name of Pythagoras was Lucky's saviour. She used her wonderful theorem to work out the height of her triangle before working out the area. Brilliant!



So, Lucky worked and worked through sheets and sheets of her mathematical calculations until one day she found her Isosceles triangle design! It was a big sense of achievement for Lucky.

Lucky found out that $19 + 19 + 18 = 56$, this gives her the largest area.

$18 - 9 = 9$
 $h = \sqrt{19^2 - 9^2}$
 $= 16.7$
 18 Area $= \frac{1}{2} \times 18 \times 16.7$
 $= 150.3 \text{ m}^2$

At 12 noon on a Sunday afternoon, the queen asked all four cats to come to the palace with their garden design. The pile of 1m wooden fencing was laid out on the marble floor ready for the queen's servants to carry out into the yard to build the garden area. This garden is for little kittens to play in.



'Bluestar' she said, 'show me your design'.

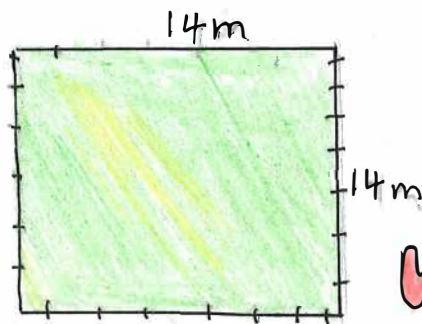
'Your majesty, I went for the circular design' and she presented it to the queen with her head held high. 'Your kittens will enjoy an area of 254 m^2 '. 'I see' remarked the queen.

254 m^2

Yay

"Will I Win?"

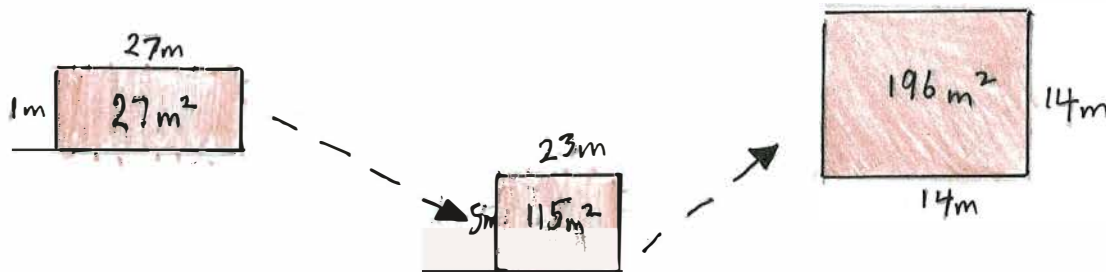
'Paws, your design please'. 'I ...I... went for the square design your majesty' Paws replied nervously. 'It took me rather a long time but it does give your kittens an area $196m^2$ to play in' Paws said proudly.



$4 \times 14m = 56m \rightarrow$ perimeter
 Area: $14 \times 14 = 196m^2$

Would
 PAWS
 win?

"How about you, Sparkle?" The queen asked. "Your design" the queen ordered. 'Well to be honest, my rectangular design ended up being a square' Paws looked over to Sparkle and smirked. 'It took me a very long time and it does give your kittens an area of $196m^2$ ' paws said in a very competitive way.



"It's over to you now Lucky. What have you got to show me" said the queen impatiently. After building up enough courage, Lucky delivered her design to the queen. "I went for the Isosceles triangle design" Lucky bravely said. She waved her ingenious calculations in front of the queen's face and said: "Your kittens will enjoy an area of approximately $150m^2$." Lucky's voice echoed around the room. There was a complete silence before the queen announced the next cat to take the throne. And we know who that is don't we.

The
 End

There once were 4 cats named: Lucky, Bluestar, Paws and Sparkle.

They were very good friends that worked, played and collaborated together. Their hobbies was to hunt in the evenings, when all the animals were asleep. They loved to watch the moon fade away in the early hours in the morning. That was why they were called the "Moonlight clan". The queen chose them four cats, because they were the smartest cats in the whole country. The four cats were set a task by the queen to make the biggest area of their garden. Whoever make the largest garden gets to be the next queen, there was one winner...

About the author

Hi! I am Catalina Liu from Dulwich college Beijing. I

am 12 years old this year and I love cats so much!

I have a pet cat at home that is white. The idea for this story that I thought of was when I decided to take my cat to play in the garden and wanted to know how big this garden was.

That time I had this idea for this story. I used

4 characters and 4 types of shapes in this story.

Maths is everywhere, it is very useful in our life.

